Pre Calculus GT
Final Review Answers

① $(x-h)^2+(y-k)^2=r^2$

$(-2-1)^2+(3+1)^2=r^2$

$9+16=r^2$

$25=r^2$

$\boxed{(x-1)^2+(y+1)^2=25}$

② $x^2-4x+y^2+6y=-3$

$x^2-4x+4+y^2+6y+9=-3+4+9$

$(x-2)^2+(y+3)^2=10$

$\boxed{C(2,-3)\quad r=\sqrt{10}}$

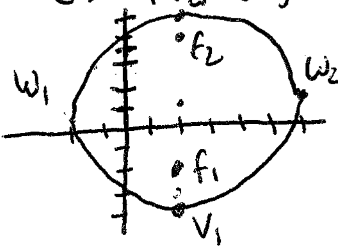
③ $\frac{(x-2)^2}{16}+\frac{(y-1)^2}{25}=1$

$C(2,1)$ majoraxis: y-axis

$a^2=25$ $V(2,1\pm5)$

$b^2=16$ $W(2\pm4,1)$ $e=\frac{3}{5}$

$c^2=9$ v_2 $F(2,1\pm3)$



$0<e<1$ forall ellipses

$PF_1+PF_2=2a$

$=2(5)=10$

④ $C(2,2)$ $f(2,6)$ $V(2,7)$

$c=4$ $c^2=16$

$a=5$ $a^2=25$

major axis: y-axis

$\boxed{\frac{(y-2)^2}{25}+\frac{(x-2)^2}{9}=1}$

⑤ $10x^2+5y=0$

$10x^2=-5y$ $x^2=-\frac{1}{2}y$

$V(0,0)$ ⌒

A of S: $x=0$

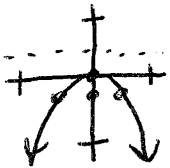
$4p=-\frac{1}{2}$ $F(0,-\frac{1}{8})$

$p=-\frac{1}{8}$

D: $y=\frac{1}{8}$

L.R.: $2p=\frac{1}{4}$

$(\frac{1}{4},-\frac{1}{8})\ (-\frac{1}{4},-\frac{1}{8})$



True

⑥ $x^2+4x-4y^2+32y=64$

$x^2+4x+4-4(y^2-8y+16)=64+4-4(16)$

$(x+2)^2-4(y-4)^2=4$

$\frac{(x+2)^2}{4}-\frac{(y-4)^2}{1}=1$

$C(-2,4)$ t.a. x-axis

$a^2=4$ $V(-2\pm2,4)$

$b^2=1$ $W(-2,4\pm1)$

$c^2=5$ $F(-2\pm\sqrt{5},4)$

$e=\frac{\sqrt{5}}{2}$ Asy: $y-4=\frac{\pm1}{2}(x+2)$

Hyperbola $e>1$ $PF_1-PF_2=2a$

$=2(2)=4$

⑦ a) opens ⊂

$(y-k)^2=4p(x-h)$

$(y-2)^2=16(x+2)$

b) major: x-axis $2c=8$

$2a=10$ $c=4$

$a=5$ $c^2=16$

$a^2=25$ $b^2=9$

$C(-2,7)$

$\frac{(x+2)^2}{25}+\frac{(y-7)^2}{9}=1$

⑧ $-3\,\rfloor\; 4\quad 3\quad 0\quad 1\quad -7\quad 9$

$-12\quad 27\quad -81\quad 240\quad -69$

$4\quad -9\quad 27\quad -80\quad 233\quad -69$

$\boxed{f(-3)=-690}$

⑨
$$\begin{array}{r} 3x^3-2x^2-x+4 \\ 5x+3\,\overline{)\,15x^4-x^3-11x^2+17x+9} \\ \underline{15x^4+9x^3} \\ -10x^3-11x^2 \\ \underline{-10x^3-6x^2} \\ -5x^2+17x \\ \underline{-5x^2-3x} \\ 20x+9 \\ \underline{20x+12} \\ -3 \end{array}$$

$3x^3-2x^2-x+4+\frac{-3}{5x+3}$

⑩ $f(x)=x^4+2x^3-7x^2-8x+12$

use TI to find 1 then 2

$1\rfloor\; 1\quad 2\quad -7\quad -8\quad 12$

$1\quad 3\quad -4\quad -12$

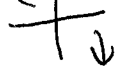
$2\rfloor\; 1\quad 3\quad -4\quad -12\quad \boxed{0}$

$2\quad 10\quad 12$

$1\quad 5\quad 6\quad \boxed{0}$

x^2+5x+6

$\boxed{(x+2)(x+3)(x-1)(x-2)}$

⑪ 

T.P.: 4 or 2 or 0

⑫ $y(x)=x^3+2x^2-5x-6$

y-int: $(0,-6)$

x-int: $\begin{array}{r|rrrr} \underline{2} & 1 & 2 & -5 & -6 \\ & & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & \boxed{0} \end{array}$

$x^2+4x+3=0$

$(x+3)(x+1)(x-2)$

$x=-3,-1,2$

⑬ $(x-1)(x+2)(x-3)=f(x)$

$(x-1)(x^2-x-6)=f(x)$

$x^3-x^2-6x-x^2+x+6$

$\boxed{x^3-2x^2-5x+6=f(x)}$

⑭ $f(x)=\dfrac{2x^2+4x-6}{x^2+5x+6}$

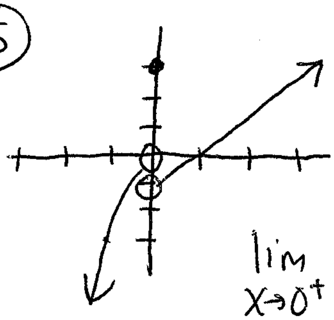
$f(x)=\dfrac{2(x+3)(x-1)}{(x+3)(x+2)}$

HA: $y=2$

VA: $x=-2$ non-removable

hole: $(-3,8)$ removable

⑮

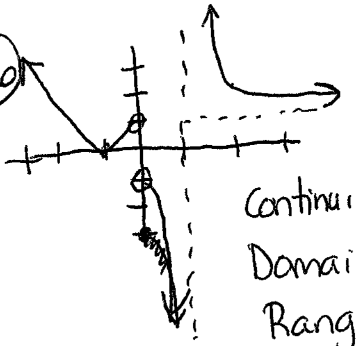


non-removable discontinuity at $x=0$

$\lim_{x\to0^+} y(x)=-1$

$\lim_{x\to0^-} y(x)=0$

⑯



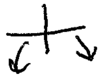
Continuity: $(-\infty,0)(0,1)(1,\infty)$

Domain: $(-\infty,1)(1,\infty)$

Range: $(-\infty,-1)\ [0,\infty)$

⑰ Domain $(-\infty,\infty)$

Range: $y=K$; K is an integer

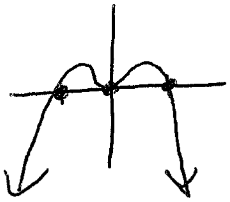
⑳ $y=x^2-x^4$ end 

y-int: $(0,0)$

x-int: $x^2(1-x^2)$

$x=0,\pm1$

double



㉑ $f(x)=\dfrac{x(x-2)}{x-1}$

holes: none

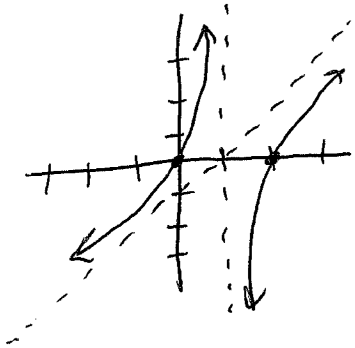
VA: $x=1$

HA: none

y-int $(0,0)$

x-int $(0,0)(2,0)$

OA: $\begin{array}{r|rrr} \underline{1} & 1 & -2 & 0 \\ & & 1 & -1 \\ \hline & 1 & -1 & -1 \end{array}$ $y=x-1$



㉒ a) $v(t)=-3\sin t$

$a(t)=-3\cos t$

b) $\dfrac{v(\frac{\pi}{4})-v(0)}{\frac{\pi}{4}-0}=\dfrac{-\frac{3}{\sqrt2}-0}{\frac{\pi}{4}}=\dfrac{-12}{\pi\sqrt2}$

c) Change to speed incr or decr at $t=\frac{\pi}{4}$ and $t=\frac{7\pi}{4}$

$v(\frac{\pi}{4})<0$ $a(\frac{\pi}{4})<0$ so speed is increasing

$v(\frac{7\pi}{4})>0$ $a(\frac{7\pi}{4})<0$ so speed is decreasing

㉓ a) $f'(x)=(5x^3-3)(8x)+(4x^2+3)(15x^2)$

$=40x^4-24x+60x^4+45x^2$

$=100x^4+45x^2-24x$

b) $f'(x)=\dfrac{(4x^2+3)(15x^2)-(5x^3-3)(8x)}{(4x^2+3)^2}$

$=\dfrac{60x^4+45x^2-40x^4+24x}{(4x^2+3)^2}$

$=\dfrac{20x^4+45x^2+24x}{(4x^2+3)^2}$

㉓ c) $h'(x)=20x^{\frac{3}{2}}-\frac{9}{2}x^{\frac{1}{2}}-19$ d) $y'(x)=2\cos x-\pi\sin x$ e) 0

㉔ $f(x)=2x^2-3x+1$

a) $f'(x)=4x-3$

$f'(1)=4-3=1$

$f(1)=0$

$y-0=1(x-1)$

$y=x-1$

b) $f'(-2)=-11$

$f(-2)=15$

$y-15=\frac{1}{11}(x+2)$

㉖ $\lim\limits_{x\to -4}\frac{x^3+64}{x+4}$

$\lim\limits_{x\to -4}\frac{(x+4)(x^2-4x+16)}{x+4}$

$\lim\limits_{x\to -4} x^2-4x+16=48$

㉗ $\lim\limits_{x\to 12}\frac{\sqrt{x-3}-3}{x-12}\ \frac{\sqrt{x-3}+3}{\sqrt{x-3}+3}$

$\lim\limits_{x\to 12}\frac{x-3-9}{(x-12)(\sqrt{x-3}+3)}$

$\lim\limits_{x\to 12}\frac{x-12}{(x-12)(\sqrt{x-3}+3)}$

$\lim\limits_{x\to 12}\frac{1}{\sqrt{x-3}+3}=\frac{1}{6}$

㉘ $\lim\limits_{x\to 0}\frac{\sin\frac{1}{5}x}{x}\cdot\frac{\frac{1}{5}}{\frac{1}{5}}$

$\lim\limits_{x\to 0}\frac{1}{5}\frac{\sin\frac{1}{5}x}{\frac{1}{5}x}$

$\frac{1}{5}$

㉙ $\lim\limits_{x\to -2}\frac{x^2-4}{x+2}=\lim\limits_{x\to -2}\frac{(x+2)(x-2)}{x+2}$

$=-4$

$\lim\limits_{x\to -2}\frac{x-2}{\cos(x+2)}=\frac{-4}{1}=-4$

So $\lim\limits_{x\to -2}f(x)=-4$ by the Squeeze (Sandwhich) Theorem

㉚ $\lim\limits_{\Delta x\to 0}\frac{\frac{2}{x+\Delta x}-\frac{2}{x}}{\Delta x}\ \frac{x(x+\Delta x)}{x(x+\Delta x)}$

$\lim\limits_{\Delta x\to 0}\frac{2x-2(x+\Delta x)}{x\Delta x(x+\Delta x)}$

$\lim\limits_{\Delta x\to 0}\frac{-2\cancel{\Delta x}}{x\cancel{\Delta x}(x+\Delta x)}=\frac{-2}{x^2}$

㉛ (i) $h(2)=2$

(ii) $\lim\limits_{x\to 2^-}h(x)=0$

$\lim\limits_{x\to 2^+}h(x)=0$

$\lim\limits_{x\to 2}h(x)=0$

(iii) $h(2)\neq\lim\limits_{x\to 2}h(x)$

therefore h is disc. at $x=2$

㉜ $\frac{(3x+1)^{-\frac{1}{2}}(x-2)^{-\frac{2}{3}}[3x+1+x-2]}{(x-2)^{\frac{1}{3}}(3x+1)^{\frac{1}{2}}}$

$(3x+1)^{-1}(x-2)^{-1}[4x-1]$

$\frac{4x-1}{(3x+1)(x-2)}$

㉝ $\sin 2x=-\cos x$

$2\sin x\cos x=-\cos x$

$2\sin x\cos x+\cos x=0$

$\cos x(2\sin x+1)=0$

$\cos x=0\quad \sin x=-\frac{1}{2}$

$x=\frac{\pi}{2},\frac{3\pi}{2},\frac{7\pi}{6},\frac{11\pi}{6}$

㉞ $2\sin^2x=1+2\sin x$

$2\sin^2x-2\sin x-1=0$

$\sin x=\frac{2\pm\sqrt{4-4(2)(-1)}}{2(2)}$

$=\frac{2\pm\sqrt{12}}{4}$

$=\frac{2\pm 2\sqrt{3}}{2}=1\pm\sqrt{3}=\sin x$

only use $1-\sqrt{3}$

$\sin x=1-\sqrt{3}$

$\sin^{-1}(1-\sqrt{3})=x$

$\boxed{x=-.821}$

㉟ $f(x) = \begin{cases} \cos x & x \leq \frac{\pi}{2} \\ \sin x & |x-\pi| < \frac{\pi}{2} \\ 3 & x > \frac{3\pi}{2} \end{cases} \rightarrow$ $x-\pi < \frac{\pi}{2}$ $\quad$ $x - \pi > -\frac{\pi}{2}$

$x < \frac{3\pi}{2}$ $\quad$ $x > \frac{\pi}{2}$

$\frac{\pi}{2} < x < \frac{3\pi}{2}$

i) $f(\frac{3\pi}{2})$ undefined so...

f is discontinuous at $x = \frac{3\pi}{2}$

(Don't need to finish triple test since (i) doesn't work

㊱ $V = <-3, 3>$

Magnitude: $||\vec{v}|| = \sqrt{(-3)^2 + (3)^2} = \sqrt{18} = \boxed{3\sqrt{2}}$

(Norm)

Direction: $\cos\alpha = \frac{-3}{3\sqrt{2}}$

$\cos\alpha = -\frac{1}{\sqrt{2}}$

2nd quad so $\boxed{\alpha = 135°}$

㊲ $<5, -8> <4, -2>$

$5(4) + -8(-2)$

$36 \neq 0$ so not perpendicular

$<5, -8> <2, 4>$

$5(2) + -8(4)$

$-22 \neq 0$ so not parallel

So $\boxed{\text{neither}}$

㊳ $||\hat{v}|| = 6$

$\alpha = -120°$

$x = 6\cos -120 = 6(-\frac{1}{2}) = -3$

$y = 6\sin -120 = 6(-\frac{\sqrt{3}}{2}) = -3\sqrt{3}$

$\boxed{<-3, -3\sqrt{3}>}$