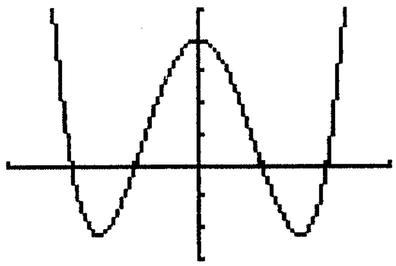
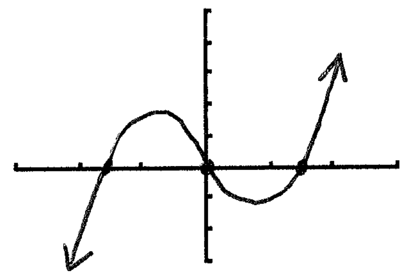
Precalculus GT Review Sheet for Test on All of Derivatives.

Name Answers

I. The graph of f (x) is given. Sketch the derivative of f on the grid at right.





II. Answer the following. Show work and box in your answer.

1. Use the definition of derivative to find $f'(x)$ if $f(x)=\sqrt{2x+3}$.

$$\lim_{h\to 0}\frac{\sqrt{2x+2h+3}-\sqrt{2x+3}}{h}\cdot\frac{\sqrt{2x+2h+3}+\sqrt{2x+3}}{\sqrt{2x+2h+3}+\sqrt{2x+3}}$$

$$\lim_{h\to 0}\frac{2x+2h+3-2x-3}{h(\sqrt{2x+2h+3}+\sqrt{2x+3})}$$

$$\lim_{h\to 0}\frac{2}{\sqrt{2x+2h+3}+\sqrt{2x+3}}$$

$$\frac{2}{2\sqrt{2x+3}}=\boxed{\frac{1}{\sqrt{2x+3}}}$$

2. Use the Alternate Form of the Derivative to find $f'(5)$, if $f(x)=\frac{10}{x}$

$$\lim_{x\to 5}\frac{\frac{10}{x}-2}{x-5}\ \frac{x}{x}$$

$$\lim_{x\to 5}\frac{10-2x}{x(x-5)}$$

$$\lim_{x\to 5}\frac{2(5-x)}{x(x-5)}$$

$$\lim_{x\to 5}\frac{-2}{x}=\boxed{\frac{-2}{5}}$$

3. If $f(x)=\begin{cases}2x^2-3x+1, & x\le 0\\ 5-x^3, & x>0\end{cases}$, show that f (x) is not differentiable at x = 0, using one-sided derivatives. Write a conclusion telling how your work shows this.

left side derivative: $4x-3$ at $x=0$; -3

right side derivative: $-3x^2$ at $x=0$; 0

Since left side derivative does not equal right side derivative

The derivative does not exist at $x=0$

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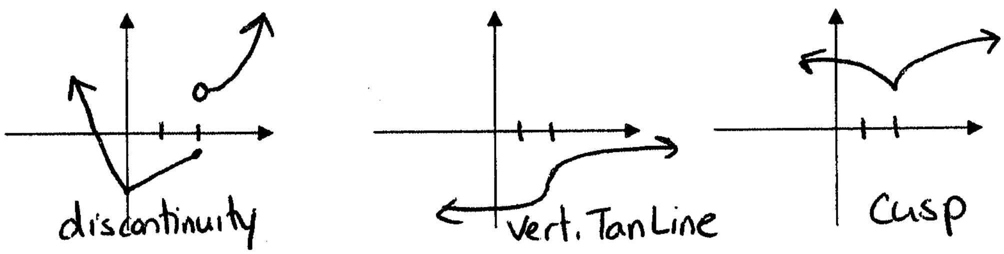
4. **Give the interval(s) over which** $f(x) = \begin{cases} \sqrt{x} & , 0 \le x \le 4 \\ x - 4 & , x > 4 \end{cases}$ **is continuous and differentiable.**

$\lim_{x \to 4^-} f(x) = \sqrt{4} = 2 \quad \lim_{x \to 4^+} f(x) = 4 - 4 = 0$

$f'(4)^- = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}(4)^{-\frac{1}{2}} = \frac{1}{4}$

Continuous $(0,4)\ (4,\infty)$ **Differentiable** $(0,4)\ (4,\infty)$ $\quad f'(4)^+ = 1$

5. **There are several reasons that derivatives fail to exist at a single point. In the space below, sketch 3 functions whose derivatives fail to exist at x = 2, for three different reasons.**



6. **Suppose that u and v are differentiable at x = 5. If u (5) = -7, u ' (5) = 2, v (5) = -8, and v ' (5) = -3 , find the following.**

a. $\frac{d}{dx}[uv]$

$uv' + vu'$

$-7(-3) + -8(2)$

$21 - 16$

5

b. $\frac{d}{dx}\left[\frac{v}{u}\right]$

$\frac{uv' - vu'}{u^2}$

$\frac{-7(-3) - (-8)(2)}{(-7)^2} = \frac{21+16}{49} = \frac{37}{49}$

c. $\frac{d}{dx}\left[\frac{3u}{5v}\right]$

$\frac{5v(3u') - 3u(5v')}{25v^2}$

$\frac{5(-8)(3(2)) - 3(-7)(5)(-3)}{25(-8)^2}$

$\frac{-240 - 315}{1600} = \frac{-555}{1600} = \frac{-111}{320}$

7. **In the table below, s (t) is position in feet, v (t) is velocity, a (t) is acceleration, and t is in seconds, for a particle in motion over [0 , 2.4]. Show work to the problems below.**

t	0	.4	.8	1.2	1.6	2.4
s (t)	4	3.226	1.210	-1.126	-2.246	8.376
v (t)	0	-3.744	-5.592	-5.088	.384	31.296
a (t)	-10	-8.08	-2.32	7.28	20.72	59.12

a. **Give the average velocity of the particle from t = .8 sec to t = 2.4 sec.**

$\frac{s(2.4) - s(.8)}{2.4 - .8}$

$\frac{8.376 - 1.210}{1.6}$

4.479 ft/sec

b. **Give an approximation for v ' (1).**

$\frac{v(1.2) - v(.8)}{1.2 - .8}$

$\frac{-5.088 + 5.592}{.4}$

1.26 ft/sec^2

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7. (continued)

c. Is the speed of the particle increasing or decreasing at t = 1.2 seconds? At t = .4 seconds. Justify your answers.

At $t=1.2$ sec $v(t)<0, a(t)>0$ So speed is decreasing
At $t=.4$ sec $v(t)<0, a(t)<0$ So speed is increasing

8. How fast is the volume (V) of a cube changing with respect to the length of its side (s), when the length of its side is s = 4 inches? Show work.

$V=s^3$
$V'=3s^2$
$V'=3(4)^2=48 in^3$

9. How fast is the area of a circle changing, with respect to its radius, when the radius is r = 4 feet? Show work.

$A=\pi r^2$
$A'=2\pi r$
$A'(4)=2\pi(4)=8\pi ft^2$

III. Differentiate. Show work and box in your answers.

1. $y = 5x^3 - 2x^2 + \pi^5 x - 7$

$y' = 15x^2 - 4x + \pi^5$

2. $f(x) = 6x^{\frac{5}{3}} - 12x^{\frac{2}{3}} + 3\sqrt[3]{x}$

$f'(x) = 10x^{\frac{2}{3}} - 8x^{-\frac{1}{3}} + x^{-\frac{2}{3}}$

$f'(x) = 10x^{\frac{2}{3}} - \frac{8}{x^{\frac{1}{3}}} + \frac{1}{x^{\frac{2}{3}}}$

3. $y(x) = 3\sin x - 2\cos x$

$y'(x) = 3\cos x + 2\sin x$

4. $y = \dfrac{2x+1}{x-2}$

$y' = \dfrac{(x-2)(2)-(2x+1)(1)}{(x-2)^2}$

$y' = \dfrac{2x-4-2x-1}{(x-2)^2}$

$y' = \dfrac{-5}{(x-2)^2}$

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5. $g(x) = \dfrac{3x^4 - 2x^3 + 5x^2 - 3x}{x^2} = 3x^2 - 2x + 5 - 3x^{-1}$

$g'(x) = 6x - 2 + 3x^{-2}$

$g'(x) = 6x - 2 + \dfrac{3}{x^2}$

6. $y = x^2 \tan x$

$y' = x^2 \sec^2 x + 2x \tan x$

7. $y(x) = x \sin x + x \cos x$

$y'(x) = x\cos x + \sin x + x(-\sin x) + \cos x$

$y'(x) = x\cos x + \sin x - x\sin x + \cos x$

8. $h(x) = \sqrt{x} \sec x = x^{\frac{1}{2}} \sec x$

$h'(x) = x^{\frac{1}{2}} \sec x \tan x + \frac{1}{2} x^{-\frac{1}{2}} \sec x$

$h'(x) = x^{\frac{1}{2}} \sec x \tan x + \dfrac{\sec x}{2x^{\frac{1}{2}}}$

9. $y = 5 \cot x$

$y' = -5\csc^2 x$

10 $f(x) = \dfrac{3\tan x}{x}$

$f'(x) = \dfrac{x\, 3\sec^2 x - 3\tan x}{x^2}$

$f'(x) = \dfrac{3x\sec^2 x - 3\tan x}{x^2}$

11. $y = (x^3 + 3)(2 - x^2)$

$y = 2x^3 - x^5 + 6 - 3x^2$

$y' = -5x^4 + 6x^2 - 6x$

12. $g(x) = \dfrac{2x^2 - 5x + 2}{x - 3}$

$g'(x) = \dfrac{(x-3)(4x-5) - (2x^2-5x+2)(1)}{(x-3)^2}$

$g'(x) = \dfrac{4x^2 - 17x + 15 - 2x^2 + 5x - 2}{(x-3)^2}$

$g'(x) = \dfrac{2x^2 - 12x + 13}{(x-3)^2}$

13. $y = x^{-3} + 2x^{-2} - \dfrac{3}{x}$

$y' = -3x^{-4} - 4x^{-3} + 3x^{-2}$

$y' = \dfrac{-3}{x^4} - \dfrac{4}{x^3} + \dfrac{3}{x^2}$

14. $f(x) = (\tan x)(\cos x)$

$f'(x) = \tan x(-\sin x) + \cos x \sec^2 x$

$f'(x) = -\tan x \sin x + \cos x \sec^2 x$